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# Comment on the hadronic decay of excited heavy quarkonia

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**Abstract.** We make comments on [1], and provide partial wave analysis to the decays of excited heavy S-wave $1^-$ quarkonia into the basic $1^-$ quarkonia state plus $\pi\pi$. It is revealed that there exist contributions of D-wave transition in $\psi' \longrightarrow J/\psi\pi\pi$, $\Upsilon(2S) \longrightarrow \Upsilon(1S)\pi\pi$ and $\Upsilon(3S) \longrightarrow \Upsilon(1S)\pi\pi$ by using the data-fitting results in [1]. A possible experimental method to measure the D-wave directly is discussed.

## 1 Introduction

Starting from the infinite mass limit for the heavy quarkonium, the authors of [1] have presented an interesting systematic derivative expansion for the decays of a heavy excited S-wave spin-1 quarkonium into a lower S-wave spin-1 state in the spirit of chiral perturbation theory. An effective Lagrangian for these exclusive hadronic decays has been constructed in [1]. It is of as follows

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rm SB} \tag{1}$$

$$\begin{aligned}\mathcal{L}_0 = {} & g A_\mu^{(v)} B^{(v)\mu *} \mathrm{Tr}\left[(\partial_\nu U)\,(\partial^\nu U)^\dagger\right] \\ & + g_1 A_\mu^{(v)} B_\nu^{(v)*} \mathrm{Tr}\left[(v{\cdot}\partial U)\,(v{\cdot}\partial U)^\dagger\right] \\ & + g_2 A_\mu^{(v)} B^{(v)\mu *} \mathrm{Tr}\left[(\partial^\mu U)\,(\partial^\nu U)^\dagger \right. \\ & \left. + (\partial^\mu U)^\dagger (\partial^\nu U)\right] + h.c. \end{aligned} \tag{2}$$

$$\begin{aligned}\mathcal{L}_{\rm SB} = {} & g_3 A_\mu^{(v)} B^{(v)\mu *} \mathrm{Tr}\left[M(U + U^\dagger - 2)\right] \\ & + i g' \varepsilon^{\mu\nu\alpha\beta} \left[v_\mu A_\nu^{(v)} \partial_\alpha B_\beta^{(v)*} - \partial_\mu A_\nu^{(v)} v_\alpha B_\beta^{(v)*}\right] \\ & \times \mathrm{Tr}\left[M(U - U^\dagger)\right] + h.c. \end{aligned} \tag{3}$$

where $U$ is a unitary 3×3 matrix that contains the Goldstone fields, $M$ is light-quark mass matrix, and $g$, $g_1$, $g_2$, $g_3$ and $g'$ are constants. And $A_\mu$ is the field of initial S-wave $1^-$ quarkonium state, $v$ is its velocity vector, $B_\mu$ the one of the final $1^-$ quarkonium state, and $A_\mu^{(v)}$ and $B_\mu^{(v)}$ are defined by a phase redefinition of $A_\mu$ and $B_\mu$ respectively. By using this Lagrangian, the authors of [1] have obtained a good fit to the data, especially to $\psi' \longrightarrow J/\psi\pi\pi$, $\Upsilon(2S) \longrightarrow \Upsilon(1S)\pi\pi$ and $\Upsilon(3S) \longrightarrow \Upsilon(1S)\pi\pi$. However, they do not present a correct partial wave analysis to the decay amplitudes. The decays are S-wave transition dominant, but in principle D-wave transition is not forbidden, and to search the signals of D-wave in the transitions is meaningful. We will point out in this present comment on [1] that to view the $g_1$-term in $\mathcal{L}_0$ (2) to be a S-wave transition purely is a misunderstanding. We will show that the D-wave transition signals can be seen in the processes of $\psi' \longrightarrow J/\psi\pi\pi$, $\Upsilon(2S) \longrightarrow \Upsilon(1S)\pi\pi$ and $\Upsilon(3S) \longrightarrow \Upsilon(1S)\pi\pi$. And the ratios between D-wave contributions to the transitions and the total are determined. In addition, we will also propose a possible experiment to measure the D-wave transitions directly, and will discuss how to determine all constans in the process of $A_\mu \longrightarrow B_\mu\pi\pi$ experimently besides a overall factor.

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## 2 Kinematics and partial wave analysis

For definiteness, we consider the case of $A_\mu = \psi'$ and $B_\mu = J/\psi$ in this section. In the rest frame of $\psi'$, $p_{\pi^+}$ and $p_{\pi^-}$ are 4-momentum of $\pi^+$ and $\pi^-$ respectively, $q = p_{\pi^+} + p_{\pi^-}$, $r = p_{\pi^+} - p_{\pi^-}$ and the partial decay rate of $\psi'$ into $J/\psi\pi^+\pi^-$ is given by

$$d\Gamma = A_{\rm ph} \left|\mathcal{M}\right|^2 dm_{\pi\pi} d\cos\theta \tag{4}$$

where $\mathcal{M}$ is the decay amplitude,

$$\begin{aligned} A_{\rm ph} &= \frac{1}{2\pi^2}\frac{1}{16 m_{J/\psi}^2}(q^2 - 4m_\pi^2)^{\frac{1}{2}} \left|\vec{q}\right| \\ \left|\vec{q}\right| &= \frac{1}{2m_{\psi'}}\left[(m_{\psi'}^2 - (m_{\pi\pi} + m_{J/\psi})^2)\right. \\ & \quad \left. \times (m_{\psi'}^2 - (m_{\pi\pi} - m_{J/\psi})^2)\right]^{\frac{1}{2}} \\ q^2 &= m_{\pi\pi}^2 = (p_{\pi^+} + p_{\pi^-})^2, \end{aligned}$$

and $\theta$ is the angle between 3-momentum of $\pi^+$ and 3-momentum of $J/\psi$ in the rest frame of $\pi^+ - \pi^-$. We call $\theta$

as correlation angle hereafter. From the Lagrangian of (1), one can get the transition amplitude $\mathcal{M}$ to the process of $A_\mu = \psi'$ and $B_\mu = J/\psi$ as follows

$$\begin{aligned}\mathcal{M}(\psi' \to J/\psi\pi^+\pi^-) \\ = -\frac{4}{F_0^2}\left\{\left[\frac{g}{2}(m_{\pi\pi}^2 - 2M_\pi^2) + g_1(v\cdot p_{\pi^+})(v\cdot p_{\pi^-}) + g_3 M_\pi^2\right]\right. \\ \left.\times\, \varepsilon_\psi^*\cdot\varepsilon_{\psi'} + g_2[p_{\pi^+\mu}p_{\pi^-\nu} + p_{\pi^+\nu}p_{\pi^-\mu}]\varepsilon_{J/\psi}^{*\mu}\varepsilon_{\psi'}^{\nu}\right\} \quad (5)\end{aligned}$$

where $\varepsilon_{J/\psi}, \varepsilon_{\psi'}$ are the polarization vectors of $J/\psi$ and $\psi'$ respectively.

The authors of [1] argued that the $g_2-$terms in $\mathcal{M}$ (5) are strongly suppressed by the chiral symmetry breaking scale $\Lambda_{\chi SB}$ or heavy quark mass according to their derivative expansion treatment, and they then set $g_2 = 0$. This argument is consistent with the results achieved in [2], [3] and [4] based on the multipole expansion hypothesis for the soft gluon field emission from heavy quarkonium. However this does not mean the D-wave contribution has been excluded. And to view this as pion-D-wave contribution suppression is inadequate, because the $g_1-$term has D-wave component also. We show this point in follows.

In the rest frame of $\psi'$, $v = (1, 0)$

$$(v\cdot p_{\pi^+})(v\cdot p_{\pi^-}) = p_{\pi^+}^0 p_{\pi^-}^0$$

Through elementary calculations we get

$$p_{\pi^+}^0 p_{\pi^-}^0 = A(q^2)P_0(\cos\theta) + B(q^2)P_2(\cos\theta)$$

where

$$A(q^2) = \frac{1}{4}q^2 + \frac{1}{6}\left|\vec{q}\right|^2\left(1 + \frac{2m_\pi^2}{q^2}\right)$$

$$B(q^2) = -\frac{1}{2}\left|\vec{q}\right|^2\left(1 - \frac{4m_\pi^2}{q^2}\right)$$

$P_0(\cos\theta) = 1,\ P_2(\cos\theta) = \frac{1}{2}\left(\cos^2\theta - \frac{1}{3}\right)$ are Legendre functions. Thus the decay amplitude $\mathcal{M}$ (5) can be decomposed into two parts: S-wave($\mathcal{M}_\mathrm{S}$) and D-wave($\mathcal{M}_\mathrm{D}$). We have

$$\mathcal{M} = \mathcal{M}_\mathrm{S} + \mathcal{M}_\mathrm{D} \quad (6)$$

where

$$\mathcal{M}_\mathrm{S} = \mathcal{M}_0\left\{q^2 - c_1\left(q^2 + \left|\vec{q}\right|^2\right)\left(1 + \frac{2m_\pi^2}{q^2}\right) + c_2 m_\pi^2\right\} \quad (7)$$

$$\mathcal{M}_\mathrm{D} = \mathcal{M}_0\left\{3c_1\left|\vec{q}\right|^2\left(1 - \frac{4m_\pi^2}{q^2}\right)\right\}P_2(\cos\theta) \quad (8)$$

with

$$\mathcal{M}_0 = \mathrm{const.}\times\left(\varepsilon_{\psi'}\cdot\varepsilon_{J/\psi}\right)$$

$$c_1 = -\frac{g_1}{3g}\left(1 + \frac{g_1}{6g}\right)^{-1}$$

$$c_2 = 2\left(\frac{g_3}{g} - \frac{g_1}{3g} - 1\right)\left(1 + \frac{g_1}{6g}\right)^{-1}$$

The ratio of D-wave transition rate to the total decay rate is defined by

$$\mathcal{R}_\mathrm{D} = \frac{\int dq^2 \int_{-1}^{+1} d\cos\theta \frac{1}{m_{\pi\pi}} A_\mathrm{ph} \sum_{\varepsilon\varepsilon'} \left|\mathcal{M}_\mathrm{D}\right|^2}{\int dq^2 \int_{-1}^{+1} d\cos\theta \frac{1}{m_{\pi\pi}} A_\mathrm{ph} \sum_{\varepsilon\varepsilon'} \left|\mathcal{M}_\mathrm{S} + \mathcal{M}_\mathrm{D}\right|^2} \quad (9)$$

where the limits of $q^2$ in the integrals are $q_\mathrm{min}^2 = 4m_\pi^2$, $q_\mathrm{max}^2 = (M_{\psi'} - M_{J/\psi})^2$ and the data used in the calculations are $M_{\psi'} = 3686.0MeV$, $M_{J/\psi} = 3096.88MeV$, $M_\pi = 139.57MeV$ and $\sum_{\varepsilon\varepsilon'}$ means to sum both up $\varepsilon_{J/\psi}$ and up $\varepsilon_{\psi'}$. Thus as long as $g_1$ and $g_3$ be fixed by fitting the experimental invariant mass spectrum, the contributions of D-wave to the transitions will be determined.

The extensions of the above formulas to the excited $\Upsilon$-decays are straightforward.

## 3 Ratios of D-wave transition rate to the total decay rate

From (4) and (5), we obtain the invariant $\pi-\pi$-mass spectrum

$$\begin{aligned}\frac{d\Gamma}{dm_{\pi\pi}} &= \int_{-1}^{1} d\cos\theta A_{ph} \sum_{\varepsilon\varepsilon'} |\mathcal{M}|^2 \\ &= \Gamma_0 \left|\vec{q}\right| \sqrt{q^2 - 4m_\pi^2}\left\{\left[q^2 - c_1\left(q^2 + \left|\vec{q}\right|^2\right)\right.\right. \\ &\quad \left.\times\left(1 + \frac{2m_\pi^2}{q^2}\right) + c_2 m_\pi^2\right]^2 \\ &\quad \left.+\frac{1}{5}{c_1}^2\left|\vec{q}\right|^4\left(1 - \frac{4m_\pi^2}{q^2}\right)^2\right\}. \quad (10)\end{aligned}$$

where $\Gamma_0$ is a constant.

To $\psi' \longrightarrow J/\psi\pi\pi$ and $\Upsilon' \longrightarrow \Upsilon\pi\pi$, the energies that are available for the pions are small ($< 586MeV < m_\rho$). Therefore the fit under chiral limit, i.e., $g_3 = 0$, to these processes is legitimate. This has been done in [1]. The authors of [1] obtained

$$\left(\frac{g_1}{g}\right)_{c\bar{c}}^{\mathrm{chiral}} = -0.35 \pm 0.03, \quad \text{for } \psi' \longrightarrow J/\psi\pi^+\pi^-, \quad (11)$$

$$\left(\frac{g_1}{g}\right)_{b\bar{b}}^{\mathrm{chiral}} = -0.19 \pm 0.04, \quad \text{for } \Upsilon(2S) \longrightarrow \Upsilon(1S)\pi^+\pi^-. \quad (12)$$

Substituting (11) and (12) into (9), we obtain the ratios of D-wave transition rate to the total rate for $\psi' \longrightarrow J/\psi\pi^+\pi^-$ and $\Upsilon(2S) \longrightarrow \Upsilon(1S)\pi^+\pi^-$ respectively as follows

$$\mathcal{R}_D(\psi' \longrightarrow \psi\pi\pi) = 0.065 \pm 0.018\% \quad (13)$$

$$\mathcal{R}_D(\Upsilon(2S) \longrightarrow \Upsilon(1S)\pi\pi) = 0.0156 \pm 0.0078\%. \quad (14)$$

To $\Upsilon(3S) \longrightarrow \Upsilon(1S)\pi\pi$, the energies that are available for the pions are not small, and the pions are not soft. Thus the low energy theorem based on the chiral symmetry may be not a good approximation. Consequently, the contributions of $g_3$-term turned to be significant. $g$, $g_1$ and $g_3$ of this process have been determined [1] to be

$$\frac{g_1}{g} = -2.86 \pm 0.37, \quad \frac{g_3}{g} = 15.0 \pm 1.2.$$

(These values should be checked by a further fit of the correlation angle spectrum. To see the next section.) Then the D-wave content for this process is determined

$$\mathcal{R}_D(\Upsilon(3S) \longrightarrow \Upsilon(1S)\pi\pi) = 35.5 \pm 14.2\%. \tag{15}$$

To $\psi' \longrightarrow \psi\pi\pi$, in order to reveal the contributions of $g_3-$term of (1) (an effect due to chiral symmetry breaking), the authors have designed a fitting procedure to fit the experimental invariant $\pi-\pi$ mass spectrum. However, it is of a lack of ground in physics. We will show in the next section that the value of $g_3$ can be determined by fitting both the invariant mass spectrum and the correlation angle spectrum of this process.

## 4 Direct measurement to D-wave transition

The fact that there exist a small amount of D-wave transitions in the processes of $\psi' \longrightarrow J/\psi\pi\pi$, $\Upsilon' \longrightarrow \Upsilon\pi\pi$ etc indicates that these decays are not exactly isotropic to $\theta$-distributions. To $\psi' \longrightarrow J/\psi\pi\pi$, the $\theta$-distributions can be explored by the correlation angle spectrum of the process, which is as follows

$$\begin{aligned}\frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta} &= \int_{2m_\pi}^{M_{\psi'}-M_{J/\psi}} dm_{\pi\pi} A_{ph}|\mathcal{M}|^2 \\ &= \mathrm{const.}\Gamma_0 \Bigg(1 + 0.18\mathrm{c}_2 + 0.0085{\mathrm{c}_2}^2 - 3.54\mathrm{c}_1 \\ &\quad -0.35c_1c_2 + 3.54{c_1}^2 \\ &\quad + \left(0.77c_1 + 0.077c_1c_2 - 1.62c_1^2\right)\left(\cos^2\theta - \frac{1}{3}\right) \\ &\quad +0.20c_1^2\left(\cos^2\theta - \frac{1}{3}\right)^2\Bigg)\end{aligned} \tag{16}$$

A fit to this spectrum represents a direct measurement to the partial waves in the decay.

In the decay amplitude $\mathcal{M}$ without $g_2$-terms (6), there are two unknown parameters $g_1/g$ and $g_3/g$. They could be determined by fitting two measured curves: the invariant mass spectrum (10) and the correlation angle ($\theta$) spectrum (16). The fit of the former has been performed in [1], and fit of the latter is expected. Because the ratios of D-wave transition rate to the total are generally less than 1%, it is not easy to see the deviations of $\theta$-distribution from the isotropic $\theta$-spectrum. However, it is essential that a great quantity of $\psi'$ ($\sim 4 \times 10^6$) have been accumulated by Beijing Electron Spectrum (BES) on BEPC [5]. This will make the measured curve of $\theta$-spectrum accurate enough to exhibit the the D-wave transition effects in the decay. To $\Upsilon(3S) \longrightarrow \Upsilon(1S)\pi\pi$, it is necessary to fit this $\theta$-spectrum in order to check the corresponding result of [1].

Finally, we like to mention that so long as the number of the events are large enough, it is practicable and helpful too to fit the measured bi-variable spectrum of follows

$$\frac{d^2\Gamma}{dm_{\pi\pi}d\cos\theta} = \sum_{\varepsilon\varepsilon'} A_{ph}|\mathcal{M}|^2. \tag{17}$$

This is a two-dimensional fit. Because both $m_{\pi\pi}$ and $\theta$ are not integrated out, more interesting informations on the dynamics of the decays are left in this bi-variable spectrum. A full expression of $\mathcal{M}$ is (5) where there are three parameters, $g_1/g$, $g_2/g$ and $g_3/g$. We like to argue that a full fit to the spectrums of (10), (16) and (17) will provide useful informations for these parameters. Since $g_2$-terms in $\mathcal{M}(5)$ describe the physics beyond its leading order effects, the informations on it would be significant to the dynamics.

## 5 Summary

An interesting systematic derivative expansion for the decays of a heavy excited S-wave spin-1 quarkonium into a lower S-wave spin-1 state is presented in [1]. In this letter, we make some comments on its results. We provide a partial wave analysis to the decays of excited heavy S-wave $1^-$ quarkonia into the basic $1^-$ quarkonia plus $\pi\pi$. It is revealed that there exist contributions of D-wave transition in $\psi' \longrightarrow J/\psi\pi\pi$, $\Upsilon(2S) \longrightarrow \Upsilon(1S)\pi\pi$ and $\Upsilon(3S) \longrightarrow \Upsilon(1S)\pi\pi$ by using the data-fitting results in [1]. A possible experimental method to measure the D-wave directly is discussed. It is expected to measure the process of $\psi' \longrightarrow \psi\pi\pi$ more precisely by using the data accumulated by BES on BEPC. We argue that through measuring the invariant mass spectrum, the correlation angle spectrum and the bi-variable spectrum precisely, three parameters in the model of [1] could all be determined.

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